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Strings, junctions and stability

Avijit Mukherjee

Department of Pure Mathematics & Department of Physics, University of Adelaide
Adelaide, SA 5005. Australia
E-mail: avijit@maths.adelaide.edu.au

Subir Mukhopadhyay

Department of Physics, University of Massachusetts
Amherst, MA 01003-4525, U.S.A.
E-mail: subir@physics.umass.edu

Koushik Ray

Department of Theoretical Physics, Indian Association for the Cultivation of Science
2A & B Raja S C Mullick Road, Calcutta 700 032. India
E-mail: koushik@iacs.res.in

Abstract: Identification of string junction states of pure SU(2) Seiberg-Witten theory as B-branes wrapped on a Calabi-Yau manifold in the geometric engineering limit is discussed. The wrapped branes are known to correspond to objects in the bounded derived category of coherent sheaves on the projective line $\mathbb{P}^1$ in this limit. We identify the pronged strings with triangles in the underlying triangulated category using Π-stability. The spiral strings in the weak coupling region are interpreted as certain projective resolutions of the invertible sheaves. We discuss transitions between the spiral strings and junctions using the grade introduced for Π-stability through the central charges of the corresponding objects.

Keywords: Solitons Monopoles and Instantons, Brane Dynamics in Gauge Theories, Supersymmetry and Duality, Differential and Algebraic Geometry.

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1. Introduction

String junctions are states in the type-IIB string theory formed by more than two strings joining at a vertex conserving certain charges. Their existence is crucial for the concinnity of duality symmetries of string theories [1]. BPS string junctions preserve a quarter of the original supersymmetry (SUSY). String junction solutions have been obtained and used in a variety of situations in F-theory [2, 3], M-theory [1, 4] and string theories in ten and lower dimensions [5]–[11]. In this article we concern ourselves with such states and phenomena related to them in supersymmetric field theories, realised by D-branes in string theories [12]–[15].

The objects of interest in this article are the half-BPS dyonic states in a four dimensional $\mathsf{N}=2$ super-Yang-Mills theory (SYM) with gauge group SU(2), known as the Seiberg-Witten theory [16]. It can be viewed as the gauge theory on the world-volume of a D3-brane in the type-IIB string theory in the presence of background 7-branes. The spectrum of this theory contains dyonic states, carrying electric as well as magnetic charges under the gauge group, identified with appropriate RR-charges of the D3- and the 7-branes in the type-IIB theory, in which the Seiberg-Witten gauge theory is embedded. Certain configurations of these dyons are interpreted as junctions in which at least three strings join at a vertex, while the other ends of the strings end of the D3-brane or the 7-branes. At the vertex the electric and the magnetic charges of the strings are balanced, as in electrical circuits, rendering the configuration neutral.

In a T-dual description as states in the type-IIA theory compactified on a Calabi-Yau threefold, $\mathcal{M}$, these states arise as even-dimensional branes wrapping holomorphic cycles in $\mathcal{M}$. The Seiberg-Witten theory, thus, has another description in terms of wrapped even-dimensional branes in the type-IIA theory. Within the scope of topological theories, these are given by branes in the topological B-model, alias B-branes, wrapping holomorphic cycles in $\mathcal{M}$. A way of obtaining the Seiberg-Witten theory, known as *geometric engineering* [17], produces gauge groups by compactifying the type-IIA theory on a suitable Calabi-Yau manifold, then zooming into regions in its moduli space where the manifold degenerates and gravity decouples. The resulting gauge group depends on the specific manner of degeneration. Thus, the string junctions of the Seiberg-Witten theory are interpreted as wrapped B-branes on some suitable degenerate Calabi-Yau manifold.

Now, B-branes wrapping holomorphic cycles in $\mathcal{M}$ are described as objects in $\mathfrak{D}^{\mathfrak{b}}(\mathcal{M})$, the bounded derived category of coherent sheaves on $\mathcal{M}$ [18]. This description is valid when $\mathcal{M}$ is sufficiently large, so much so that geometric notions make sense. The branes that are stable are described in $\mathfrak{D}^{\mathfrak{b}}(\mathcal{M})$ by introducing a grade guided by the preservation of SUSY. The corresponding criterion for stability, that is, between two configurations of branes the one with a lower grade is more stable, is called Π-stability. Altering the coupling in the Seiberg-Witten theory corresponds to perambulating the Kähler-moduli space of $\mathcal{M}$, whereby the set of Π-stable objects in $\mathfrak{D}^{\mathfrak{b}}(\mathcal{M})$ mutate giving way to different sets of stable objects in different regions. In the limit appropriate for deploying geometric engineering techniques, the spectrum of dyonic states in the Seiberg-Witten theory with $N = 2$ SUSY arise from the Π-stable B-branes [19] in the weak as well as in the strong coupling regimes.

Dyons in the Seiberg-Witten gauge theory on the world-volume of a D3-brane probe are interpreted as string junctions in the type-IIB theory in which the gauge theory is embedded [20, 14]. Moreover, as one perambulates the type-IIB moduli space, the junctions go through transitions producing other junctions or strings. The latter, before ending on a 7-brane, spiral around the marginal stability line, the line of demarkation between the weak and strong coupling regions of the gauge theory [15]. Upon interpreting the dyons as the Π-stable objects in $\mathfrak{D}^{\mathfrak{b}}(\mathcal{M})$, therefore, it is expected that such phenomena can be interpreted using Π-stability. We show that this is indeed the case.

In the abelian category underlying the derived category $\mathfrak{D}^{\mathfrak{b}}(\mathcal{M})$, a dyon may be viewed as a bound state of others, at certain points in the moduli space, according to certain projective resolutions provided by short exact sequences of sheaves on $\mathcal{M}$, pulled back from the projective line $\mathbb{P}^1$. This corresponds to a junction with three prongs, one lying in the weak coupling region, and two in the other, with the vertex lying on the marginal stability line, alluded to above. This corresponds, then, to replacing a sheaf by a projective resolution, in which the original sheaf is stable in the weak coupling region, while the two sheaves constituting the resolution are stable in the strong coupling region enclosed by the MS-line. In the corresponding triangulated category the short exact sequences correspond to distinguished triangles. Thus, a three-pronged junction, possibly with multiple strings per prong, corresponds to a distinguished triangle in the triangulated category or a short exact sequence in the abelian category. The same dyon may, at some other point in the moduli space, be viewed as a string that starts on the D3-brane and ends on either of the

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7-branes, staying always outside the line of marginal stability and possibly going around it several times [14]. We refer to these as *spiral strings*. We construct the projective resolutions of the sheaves in the abelian category that correspond to these spiral strings in an iterative fashion, corresponding to the invertible sheaves of rank $n > 1$ on $\mathbb{P}^1$. In the derived category, then, one can use either of the resolutions to limn the dyons corresponding to a particular sheaf. We show, however, that there are preferred regions in the moduli space corresponding to each of the resolutions, determined by the grade of Π-stability. Passing from one such region to another correspond to transitions between strings and junctions [15].

The organisation of the article is as follows. In the next section we recall a few notions related to Π-stability and fix notations. In section 3 we briefly review the identification of stable sheaves on $\mathbb{P}^1$ as Seiberg-Witten dyons discovered earlier [19]. In section 4 we obtain the projective resolutions that correspond to the spiral strings illustrating them with a few examples. We then proceed to discuss the transitions between the spiral strings and the junctions before concluding.

2. Π-stability & dyons

Let us start by briefly recalling the notion of Π-stability. An object of the derived category $\mathfrak{D}^{\mathfrak{b}}(\mathcal{M})$ is a triangle from the underlying triangulated category,

$$A \longrightarrow B \longrightarrow C \longrightarrow A[1]\,, \tag{2.1}$$

where A, B and C are complexes of coherent sheaves on $\mathcal{M}$, and $[n]$ denotes the shift functor shifting a complex toward the left by n degrees; so $[1]$ shifts a complex by unit degree. This in turn corresponds to short exact sequences of complexes in the underlying Abelian category,

$$0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow 0\,, \tag{2.2}$$

which can also be thought of as a projective resolution of the object C. In order to define a grade on the derived category one first defines a map from the Grothendieck group of $\mathcal{M}$ to the complex plane $Z : K(\mathcal{M}) \longrightarrow \mathbb{C}$, called the *central charge* [18] and [21]–[23]. The central charge of an object E of $\mathfrak{D}^{\mathfrak{b}}(M)$ is given by

$$Z(E) = \int_{\mathcal{M}} e^{-(B+iJ)}\,\mathrm{ch}(E)\sqrt{\mathrm{Td}(\mathcal{M})}\,, \tag{2.3}$$

at the large volume asymptote, with $B + iJ$ denoting the complexified Kähler class of $\mathcal{M}$. The phase of the central charge defines a *grade*, $\varphi(E)$, for every object E as

$$\varphi(E) = -\frac{1}{\pi}\arg Z(E)\,, \qquad -2 < \varphi(E) \leq 0\,. \tag{2.4}$$

The magnitude $|Z(E)|$ furnishes the mass of the state. The criterion of Π-stability is formulated in terms of the grade φ. A physically meaningful description of branes corresponding to *all* the objects in the triangle (2.1) or the exact sequence (2.2) exists if the grades satisfy

$$\varphi(B) - \varphi(A) = 1\,, \qquad \text{with} \qquad \varphi(C) = \varphi(B)\,. \tag{2.5}$$

This embodies the condition of alignment of charges of the branes and guarantees that the corresponding solution is a BPS state in the theory preserving $\mathsf{N} = 1$ SUSY. A brane corresponding to the object C in (2.1) is said to be *stable*, *marginally stable* or *unstable* with respect to decaying into A and B if $\varphi(B) - \varphi(A) \lesseqgtr 1$, respectively. The region of the moduli space where it is marginally stable is called a *marginal stability locus.* Thus, to start with, we have a good description of topological branes on the marginal stability locus. Once the marginal stability locus is ascertained, the brane corresponding to the object C is stable or unstable with respect to decaying into A and B, as one moves off this locus in the Kähler moduli space, if $\varphi(B) - \varphi(A) - 1$ is negative or positive, respectively. This criterion is called Π-stability. In the sequel we often refer to A and B as *the components* of C, in the context of a specific triangle.

Let us now briefly summarise the interpretation of Seiberg-Witten dyons as Π-stable objects in $\mathfrak{D}^{\mathfrak{b}}(\mathcal{M})$ in the geometric engineering limit [19] thereby identifying the string junctions as objects in $\mathfrak{D}^{\mathfrak{b}}(\mathcal{M})$. We then check how the analysis of Π-stability reproduces the transitions between strings and junctions.

The pure SU(2) Seiberg-Witten theory is geometrically engineered by zooming in around the principal component of the discriminant locus of the mirror of a Calabi-Yau manifold $\mathcal{M}$, which is a degree eight hypersurface in the resolution of $\mathbb{P}^4(2,2,2,1,1)$. The sole compact part left of $\mathcal{M}$ in this limit is a curve $\mathcal{C}$ isomorphic to the complex projective line $\mathbb{P}^1$, embedded in $\mathcal{M}$; $\imath : \mathcal{C} \hookrightarrow \mathcal{M}$, $\mathcal{C} \cong \mathbb{P}^1$. The states which become massless on the discriminant locus correspond to objects in $\mathfrak{D}^{\mathfrak{b}}(\mathcal{M})$ that are pull-back of objects of $\mathfrak{D}^{\mathfrak{b}}(\mathcal{C})$, indicated by $\imath^\star$. Thus, for the purpose of studying the gauge theory, it suffices to consider $\mathfrak{D}^{\mathfrak{b}}(\mathcal{C})$, whose objects are constructed out of the structure sheaf $\mathcal{O}_{\mathcal{C}}$ and the torsion sheaf $\mathcal{O}_p$, concentrated at a point p of the projective line, $\mathcal{C}$. The spectrum of the gauge theory is envisaged as the category of stable objects in $\mathfrak{D}^{\mathfrak{b}}(\mathcal{M})$.

In order to study Π-stability, one first identifies the pull-back of $\mathcal{O}_{\mathcal{C}}$ and $\mathcal{O}_p$ as a D6-brane and a D4-brane, respectively, and associates the corresponding central charges to the corresponding complexes of coherent sheaves on $\mathcal{M}$. Thus in the large volume asymptotic

$$\varpi = Z(\imath^\star\mathcal{O}_p), \qquad \varpi_D = Z(\imath^\star\mathcal{O}_{\mathcal{C}})\,, \tag{2.6}$$

where the periods ϖ and ϖ_D are the two solutions of the Picard-Fuchs equation of $\mathcal{M}$ in the limit under consideration. This entails the identification $\mathcal{O}_{\mathcal{M}} = \imath^\star\mathcal{O}_{\mathcal{C}}$. This provides the central charge map from $\mathfrak{D}^{\mathfrak{b}}(\mathcal{M})$ to the Grothendieck group of $\mathcal{M}$, alluded to earlier, the latter being generated in the present case by the zeroth and the second cohomology groups of $\mathcal{C}$, $H^0(\mathcal{C},\mathbb{Q})$ and $H^2(\mathcal{C},\mathbb{Q})$, respectively. The asymptotic expression of the central charge is used to fix the periods ϖ and ϖ_D in (2.6) up to a constant. Furthermore, the monodromies around the points $z = \pm 1$ and $z = \infty$ fix the periods ϖ and ϖ_D to the Seiberg-Witten periods [16, 15, 24], up to an overall constant [19]. Generally, the central charge of an object E of $\mathfrak{C}(\mathcal{M})$, the category of complexes underlying the derived category $\mathfrak{D}^{\mathfrak{b}}(\mathcal{M})$, can be written as

$$Z(E) = m\varpi_D + n\varpi\,, \tag{2.7}$$

where the integers m and n are, respectively, the rank $(\dim H^0(E))$ and the degree $(\dim H^2(E))$ of E defined through the Chern character map. We can therefore denote the state corresponding to E as (m,n). The sheaves $\imath^\star\mathcal{O}_\mathcal{C}$ and $\imath^\star\mathcal{O}_p$ can then be identified as the states $(1,0)$ and $(0,1)$ respectively. The grade of E is defined by (2.4) as the phase of the central charge (2.7). These states are identified as the dyons in the Seiberg-Witten theory, with electric and magnetic charges n and m, respectively.

3. Stable sheaves and dyons

The spectrum of dyons in the Seiberg-Witten theory consists in the Π-stable objects of $\mathfrak{D}^\mathfrak{b}(\mathcal{M})$, obtained from $\mathfrak{D}^\mathfrak{b}(\mathcal{C})$ under the pull-back $\imath^\star$. Thus, in order to find the dyons we need to analyse Π-stability of the sheaves on $\mathcal{C}$ and their complexes thereof. The dyons are identified as (m,n)-strings or string junctions in the type-IIB theory [14]. As mentioned above, an analysis of Π-stability involves two steps. First, the marginal stability locus is to be obtained. Then stability as one goes off this locus is to be checked.

Thus, in order to study Π-stability of the dyons in the Seiberg-Witten theory we first need to ascertain the marginal stability locus that, in the case at hand, is but a line, which we refer to as the *MS-line* in the sequel. Let us consider the objects in (2.2). Let the central charges of the branes involved be $Z(A) = m_A\varpi_D + n_A\varpi$, $Z(B) = m_B\varpi_D + n_B\varpi$ and $Z(C) = m_C\varpi_D + n_C\varpi$, respectively, with $m_C = m_B - m_A$ and $n_C = n_B - n_A$. Then, the MS-line is given by

$$\begin{aligned} 1 &= \varphi(B) - \varphi(A) \\ &= -\frac{1}{\pi}\arg\left(\frac{m_B\varpi_D/\varpi + n_B}{m_A\varpi_D/\varpi + n_A}\right), \end{aligned} \tag{3.1}$$

which is satisfied only if ϖ_D/ϖ is a real number. Thus, the MS-line is given by

$$\arg\left(\frac{\varpi_D}{\varpi}\right) = 0\,. \tag{3.2}$$

The curve in the z-plane is homotopic to a circle (see e.g. [27] and references therein for works related to MS-line) separating the strong and weak coupling regions of the Seiberg-Witten theory, indicated by $\mathfrak{S}$ and $\mathfrak{W}$, respectively, in figure 1. The MS-line goes through the points $z = \pm 1$. The two 7-branes and the D3-brane of the type-IIB probe picture are all transverse to the z-plane. They are thus represented by points in this plane. The 7-branes are situated at $z = \pm 1$, while the D3-brane moves around in the plane. In order to test the Π-stability of the different objects of $\mathfrak{D}^\mathfrak{b}(\mathcal{M})$ off the MS-line, $\arg(\varpi_D/\varpi)$ is defined as a continuous real-valued function of z [19]. Using (2.3), $\arg(\varpi_D/\varpi)$ is fixed to be $\pi/2$ at the large volume asymptote. It increases through π on the MS-line. From the expression of the central charge in terms of the periods, (2.7), then, we can study stability of sheaves on the z-plane. The stable objects reproduce the dyonic spectrum of the Seiberg-Witten theory [16, 24] both in $\mathfrak{S}$ and $\mathfrak{W}$. This is established by considering four different cases [19].

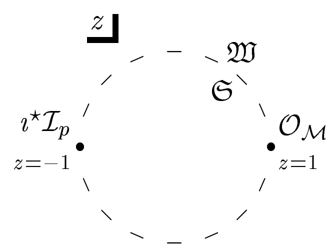

Figure 1: MS-line separating $\mathfrak{S}$ and $\mathfrak{W}$.

3.1 Ideal sheaf & structure sheaf

The two type-IIB 7-branes, with charges $(1,-1)$ and $(1,0)$, are represented by points on the z-plane and are situated at $z = \pm 1$ respectively [20]. The dyons that become massless on the MS-line are wont to stand on them provided they possess matching charges. Indeed, restricting our discussions to the upper half plane of figure 1 let us consider the pull-back to $\mathcal{M}$ of the structure sheaf $\mathcal{O}_{\mathcal{C}}$ and the ideal sheaf $\mathcal{I}_p \cong \mathcal{O}_{\mathcal{C}}(-1)$, p being the class of a point in $\mathcal{C}$. From the rank and degree of $\mathcal{I}_p$ and $\mathcal{O}_{\mathcal{C}}$ we remark that they are the dyonic states with charges $(1,-1)$ and $(1,0)$, respectively. Their central charges are

$$Z(\imath^\star \mathcal{I}_p) = \varpi_D - \varpi\,, \qquad Z(\mathcal{O}_{\mathcal{M}}) = \varpi_D\,, \tag{3.3}$$

vanishing at $z = -1$ and $z = 1$ respectively, which can be checked by explicit computation of the periods in the lower-half of the z-plane, $\mathrm{Im} z < 0$. In the upper half, on the other hand, the state corresponding to $\mathcal{O}_{\mathcal{C}}(1)$, with central charge

$$Z(\imath^\star \mathcal{O}_{\mathcal{C}}(1)) = \varpi_D + \varpi \tag{3.4}$$

becomes massless at $z = -1$. Thus, depending on whether the string approaches $z = -1$ from above or below the real axis, it is to be interpreted as $\mathcal{O}_{\mathcal{C}}(1)$ or $\mathcal{I}_p$, respectively. For the former, we shall mark the $z = -1$ point with $\mathcal{O}_{\mathcal{C}}(1)$. Recalling that the magnitude of the central charge gives the mass of the state corresponding to a sheaf, these are the two states that become massless on the MS-line. Moreover, the charges match with the charges of the 7-branes. The sheaves are marked in figure 1.

An (m,n) string corresponding to a dyon may emanate from either of these and end on the D3-brane probe with any permitted charge, situated at an arbitrary point in the z-plane, after performing sufficient charge-conserving gymnastics as we shall discuss in the next section.

3.2 Torsion sheaf

The pull-back of the torsion sheaf $\mathcal{O}_p$ is identified with the W-boson of the Seiberg-Witten theory corresponding to the dyonic state $(0,1)$. In the type-IIB probe picture the charge matches with that of the D3-brane probe. Its central charge is

$$Z(\imath^\star \mathcal{O}_p) = \varpi\,. \tag{3.5}$$

It is Π-stable in the weak coupling region $\mathfrak{W}$ but in the strong coupling region $\mathfrak{S}$ it decays into components according the the triangle

$$\mathcal{I}_p \longrightarrow \mathcal{O}_{\mathcal{C}} \longrightarrow \mathcal{O}_p \longrightarrow \mathcal{I}_p[1]\,. \tag{3.6}$$

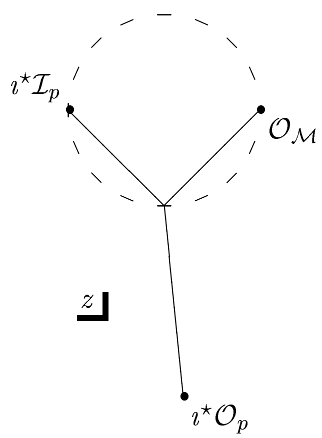

Figure 2: The W-boson.

Thus, a string emanating from the D3-brane in $\mathfrak{W}$ may bifurcate on the MS-line into $\mathcal{I}_p$ and $\mathcal{O}_{\mathcal{C}}$, which then end on the two 7-branes as discussed above in subsection 3.1. as indicated in figure 2. This provides our first example of a three-string junction [14].

3.3 Invertible sheaves on $\mathcal{C}$

The invertible sheaves on $\mathcal{C}$, namely $\mathcal{O}_{\mathcal{C}}(n)$, $n > 0$, correspond to the dyonic states $(1, n)$ in the Seiberg-Witten theory. The central charge of this dyon is

$$Z\left(\imath^{\star}\mathcal{O}_{\mathcal{C}}(n)\right) = \varpi_D + n\varpi\,. \tag{3.7}$$

These sheaves are stable in the region $\mathfrak{W}$, while decay in the region $\mathfrak{S}$ into components if $n > 1$ [19]. Thus these states can be interpreted as string junctions similar to the W-boson discussed above, but with more than one string in some of the prongs in general. However, depending on the position of the D3-brane in the z-plane an $(1, n)$ dyon can also be interpreted, using a different projective resolution, as a string spiraling the MS-line in the weak coupling region $\mathfrak{W}$ before ending on one of the 7-branes at $z = \pm 1$. We shall come back to these states in more detail in section 4.

3.4 Higher rank sheaves

A sheaf $\mathcal{E}(m, n)$ with rank m and degree n, with $m > 1$ on $\mathbb{P}^1$ splits as $\mathcal{E}(m, n) = \mathcal{O}_{\mathcal{C}}(n) \oplus \mathcal{O}_{\mathcal{C}}^{\oplus m-1}$. Such states decay on the MS-line according to

$$\begin{aligned} E(m,n) &\longrightarrow \mathcal{O}_{\mathcal{C}}(1) + (n-1)\mathcal{O}_p + (m-1)\mathcal{O}_{\mathcal{C}} \\ &\longrightarrow \mathcal{O}_{\mathcal{C}}(1) + (n-1)\mathcal{O}_{\mathcal{C}}(1) - (n-1)\mathcal{O}_{\mathcal{C}} + (m-1)\mathcal{O}_{\mathcal{C}} \\ &\longrightarrow n\mathcal{O}_{\mathcal{C}}(1) + (m-n)\mathcal{O}_{\mathcal{C}}\,, \end{aligned} \tag{3.8}$$

where we used the notation $n\mathcal{E} = \mathcal{E}^{\oplus n}$ for a sheaf $\mathcal{E}$. The decay process can be deduced as follows. On the MS-line $\mathcal{O}_{\mathcal{C}}(n)$ decays to $\mathcal{O}_{\mathcal{C}}(1)$ and $\mathcal{O}_p^{\oplus(n-1)}$ [19], since $\mathcal{O}_{\mathcal{C}}(n)$ with $n > 1$ are not stable in $\mathfrak{S}$. Hence the first line in (3.8). But nor is $\mathcal{O}_p$ stable in $\mathfrak{S}$. It decays into $\mathcal{O}_{\mathcal{C}}(1)$ and $\mathcal{O}_{\mathcal{C}}$. Combining this with the earlier decay process we deduce the decay (3.8). Both the products in the last step are stable in $\mathfrak{S}$. These states are thus represented as junctions with more than one strings connecting the vertex to the $z = \pm 1$ points as in figure 3. To summarise, the stable sheaves in the weak coupling region $\mathfrak{W}$ are $\mathcal{O}_p$ corresponding to the W-boson with dyon charge $(0, 1)$ and the invertible sheaves $\mathcal{O}_{\mathcal{C}}(n)$ with dyon charge $(1, n)$ for $n \geq 0$. In the strong coupling region $\mathfrak{S}$, however, the stable sheaves are the structure sheaf $\mathcal{O}_{\mathcal{C}}$ with dyon charge $(1, 0)$ and the invertible sheaf $\mathcal{O}_{\mathcal{C}}(1)$ with dyon charge $(1, 1)$. This corresponds to the spectra of the Seiberg-Witten theory in these two regions [16, 24, 19].

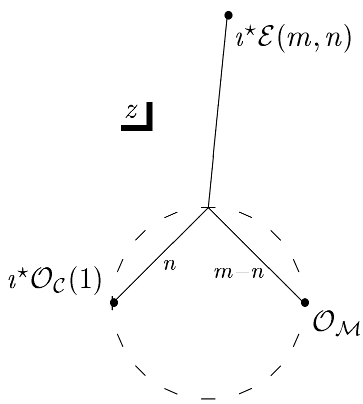

Figure 3: Higher rank sheaves as junctions.

Let us close this section with a short note on the large volume monodromy transformation of the sheaves. The large volume monodromy transformation of an object corresponds to tensoring the object in $\mathfrak{D}^b(\mathcal{C})$ with $\mathcal{O}_{\mathcal{C}}(1)$, which can be looked upon as a Fourier-Mukai transform[25]. Since coordinate of the Seiberg-Witten moduli space z is related to the type IIA moduli space through the relation $x_1 = \frac{1}{z^2}$ [19], a rotation around $z = 0$ in the z-plane would correspond to tensoring by $\mathcal{O}_{\mathcal{C}}(-2)$ in the Kähler moduli space of $\mathcal{M}$.

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4. Complexes, spirals and junctions

We already encountered an example of a three-pronged junction in subsection 3.2, where the dyon corresponding to the sheaf $\imath^\star\mathcal{O}_p$ was looked upon as a string junction bifurcating into two prongs on the MS-line. In this section we concern ourselves with the interpretation of the invertible sheaves as spiral strings and junctions. In many of the instances discussed in this section some of the prongs of a junction will have more than one strings in it, as encountered in subsection 3.4. Since we study the possible decays of an invertible sheaf $\mathcal{O}_{\mathcal{C}}(n)$, and its interpretation as junctions or spiral strings, we consider projective resolutions of it, which may replace the sheaf in the derived category. One such resolution which we shall use repeatedly is provided by the short exact sequence

$$0 \longrightarrow (n-k-1)\mathcal{O}(k) \longrightarrow (n-k)\mathcal{O}(k+1) \longrightarrow \mathcal{O}(n) \longrightarrow 0\,, \tag{4.1}$$

for $n > k+1$, n and k being integers. In the above equation as well as in the rest of this section all sheaves are on $\mathcal{C}$ with their pull-backs assumed to furnish the branes on the Calabi-Yau manifold $\mathcal{M}$. Hence we drop the subscript and $\imath^\star$ as well. Let us present a few examples first, which we shall generalise then.

4.1 Examples

Example 1. First, let us consider the invertible sheaf $\mathcal{O}(2)$ of degree 2. Putting $k=0$ and $n=2$ in (4.1) we get

$$0 \longrightarrow \mathcal{O} \longrightarrow 2\mathcal{O}(1) \longrightarrow \mathcal{O}(2) \longrightarrow 0\,. \tag{4.2}$$

In order to study the stability of $\mathcal{O}(2)$ against decaying into the components we compute the difference of their grades,

$$\varphi\big(2\mathcal{O}(1)\big) - \varphi(\mathcal{O}) - 1 = -1 - \frac{1}{\pi}\arg\left(1+\frac{\varpi}{\varpi_D}\right). \tag{4.3}$$

As we cross the MS-line the difference increases through zero. That is, $\mathcal{O}(2)$ decays into $\mathcal{O}$ and two copies of $\mathcal{O}(1)$ on the MS-line (We are not explicitly differentiating between brane and antibrane as that is obvious from the grading). The above sequence is represented as a junction with three prongs. However, now there are two strings that can end on the 7-brane at $z=-1$ corresponding to the two copies of $\mathcal{O}(1)$ in (4.2) so that the corresponding prong has two strings as shown in figure 4*a*.

Example 2. Let us now consider the degree three invertible sheaf $\mathcal{O}(3)$. Putting $k=0$ and $n=3$ in (4.1) we derive

$$0 \longrightarrow 2\mathcal{O} \longrightarrow 3\mathcal{O}(1) \longrightarrow \mathcal{O}(3) \longrightarrow 0\,. \tag{4.4}$$

On the MS-line $\mathcal{O}(3)$ decays into the components, three copies of $\mathcal{O}(3)$ and two copies of $\mathcal{O}$, governed again by (4.3). Thus this corresponds again to a three-pronged junction. Three

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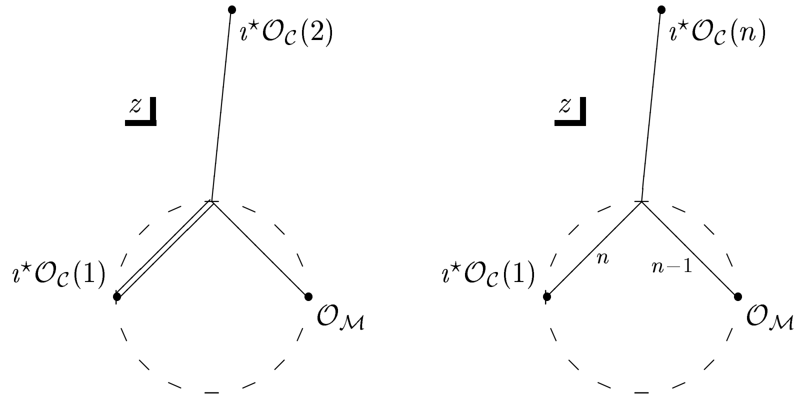


(a) $\mathcal{O}(2)$ as a junction. (b) $\mathcal{O}(n)$ as a junction.

Figure 4: Junctions

strings join the junction to the $z = -1$ point on the MS-line and two join the vertex to $z = 1$, as in figure 4b for $n = 3$.

This case presents another possibility, however. Putting $n = 3$ and $k = 1$ in (4.1) we get another projective resolution of $\mathcal{O}(3)$ given by the short exact sequence

$$0 \longrightarrow \mathcal{O}(1) \longrightarrow 2\mathcal{O}(2) \longrightarrow \mathcal{O}(3) \longrightarrow 0\,. \tag{4.5}$$

Now, in the derived category, an object may be replaced by its projective resolution, without altering its cohomology. Furthermore, the sheaf $\mathcal{O}(3)$ is Π-stable with respect to decaying into the other two sheaves appearing in the resolution (4.5). Hence, we can trade the resolution $0 \longrightarrow \mathcal{O}(1) \longrightarrow 2\mathcal{O}(2)$ for the dyon $\mathcal{O}(3)$. The string emanating from the D3-brane at an arbitrary z, corresponding to $\mathcal{O}(3)$ ends as $\mathcal{O}(1)$ on the 7-brane at $z = -1$. It goes around the MS-line once before doing so, thereby reducing its charge by undergoing a large volume monodromy transformation, corresponding to being tensored by $\mathcal{O}(-2)$, from $(1, 3)$ of $\mathcal{O}(3)$ to $(1, 1)$ of $\mathcal{O}(1)$. Hence, this corresponds to a spiral, as shown in figure 5a. Let us point out that the sheaves in the resolution are stable only in the weak coupling region $\mathfrak{W}$. Hence the spiral never crosses the MS-line.

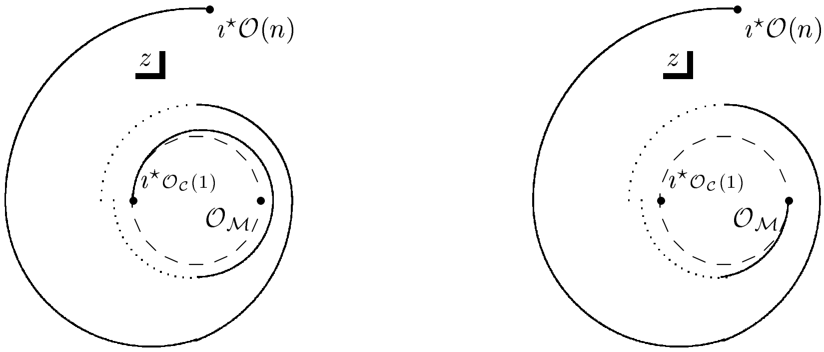
Example 3. We consider the sheaf $\mathcal{O}(4)$ with degree four next. It appears in the short exact sequence

$$0 \longrightarrow 3\mathcal{O} \longrightarrow 4\mathcal{O}(1) \longrightarrow \mathcal{O}(4) \longrightarrow 0\,, \tag{4.6}$$

obtained from (4.1) by putting $n = 4$ and $k = 0$. It decays into the components on the MS-line, governed once again by (4.3). As before, then, this corresponds to a junction as in figure 4b with four strings connecting the vertex to the 7-brane at $z = -1$ and three connecting it to the 7-brane at $z = 1$.

In order to see the spiral string corresponding to this dyon we consider another projective resolution of $\mathcal{O}(4)$. Putting $n = 4$ and $k = n - 2$ in (4.1) we obtain the short exact sequence

$$0 \longrightarrow \mathcal{O}(2) \longrightarrow 2\mathcal{O}(3) \longrightarrow \mathcal{O}(4) \longrightarrow 0\,. \tag{4.7}$$



(a) n odd: spiral string ending on $\mathcal{O}_{\mathcal{C}}(1)$. ($b$) n even: spiral string ending on $\mathcal{O}_{\mathcal{M}}$.

Figure 5: Spiral strings ending on different branes.

This, as it is can not be interpreted as a spiral, nor as a junction, since $\mathcal{O}(2)$ is not stable inside the MS-line. The resolution corresponding to a spiral must end on the left with either $\mathcal{O}$ or $\mathcal{O}(1)$, in order to be permitted to end on either 7-brane. In order to construct such a resolution we splice the sequence (4.7) with (4.2) to form the Yoneda composite

$$0 \longrightarrow \mathcal{O} \longrightarrow 2\mathcal{O}(1) \longrightarrow 2\mathcal{O}(3) \longrightarrow \mathcal{O}(4) \longrightarrow 0\ . \tag{4.8}$$

The sheaves participating in this exact sequence are stable in $\mathfrak{W}$. Hence, as before, we can trade in the projective resolution $0 \longrightarrow \mathcal{O} \longrightarrow 2\mathcal{O}(1) \longrightarrow 2\mathcal{O}(3)$ for the dyon $\mathcal{O}(4)$ in $\mathfrak{D}^{\mathfrak{b}}(\mathcal{C})$. Thus the string corresponding to the dyon $\mathcal{O}(4)$ emanating from the D3-brane in $\mathfrak{W}$ ends on the 7-brane at $z = 1$ as $\mathcal{O}$. The string spirals the MS-line twice in $\mathfrak{W}$ and never going beyond this region, thus reducing charges through large volume monodromy from $(1, 4)$ to $(1, 0)$, before ending on $\mathcal{O}$ [15], as in figure 5b.

Example 4. Let us now consider the degree five sheaf $\mathcal{O}(5)$ as our final example before we generalise these considerations. Putting $n = 5$ and $k = 0$ in (4.1) we derive the short exact sequence

$$0 \longrightarrow 4\mathcal{O} \longrightarrow 5\mathcal{O}(1) \longrightarrow \mathcal{O}(5) \longrightarrow 0\ . \tag{4.9}$$

The dyon $\mathcal{O}(5)$ decays, one more time, to the components on the MS-line governed by (4.3) and hence interpreted, as before, as a three-pronged junction as in figure 4b with $n = 5$ with five strings between the vertex and the 7-brane at $z = -1$ and four between the vertex and the 7-brane at $z = 1$.

The spiral string, once again, is obtained by considering a different projective resolution of $\mathcal{O}(5)$ in terms of sheaves stable in $\mathfrak{W}$. Putting $n = 5$ and $k = n - 2 = 3$ in (4.1) we obtain

$$0 \longrightarrow \mathcal{O}(3) \longrightarrow 2\mathcal{O}(4) \longrightarrow \mathcal{O}(5) \longrightarrow 0\ . \tag{4.10}$$

Since the sheaves $\mathcal{O}(4)$ and $\mathcal{O}(3)$ are not stable in the strong coupling region, this sequence can not be interpreted as a junction. We need to find resolutions ending with either $\mathcal{O}$ or

$\mathcal{O}(1)$ in order to obtain a spiral string. Splicing the short exact sequence (4.10) with (4.5) we derive the exact sequence

$$0 \longrightarrow \mathcal{O}(1) \longrightarrow 2\mathcal{O}(2) \longrightarrow 2\mathcal{O}(4) \longrightarrow \mathcal{O}(5) \longrightarrow 0\,. \tag{4.11}$$

The sheaves in this exact sequence are stable in $\mathfrak{W}$ and hence $\mathcal{O}(5)$ is replaced with its resolution in $\mathfrak{D}^b(\mathcal{C})$. This time, however, the string ends as $\mathcal{O}(1)$ on the 7-brane at $z = -1$, as in the case of $\mathcal{O}(3)$. The right amount of reduction of charges by large volume monodromy requires, then, that the string spirals around the MS-line four times before ending on the 7-brane.

4.2 General formulas & transitions

These considerations exemplify a general situation to which we now turn. The strategy is exactly the same as we have discussed in the context of the above examples. Namely, to obtain a three-pronged junction we need a short exact sequence which provides a resolution of a degree n invertible sheaf $\mathcal{O}(n)$ in terms of the sheaves $\mathcal{O}$ and $\mathcal{O}(1)$. Then, a spiral string is obtained as another resolution ending on the left with either $\mathcal{O}$ or $\mathcal{O}(1)$, with a sequence of sheaves in-between, all of which are stable in the weak coupling region $\mathfrak{W}$. The number of windings of the spiral around the MS-line is obtained by requiring charge conservation and remarking that the degree of a sheaf reduces by 2 under the large volume monodromy transformation. Finally, we study the transitions between the two types of resolutions.

In generalising the examples along the lines described above let us first note the projective resolution of an invertible sheaf $\mathcal{O}(n)$ of degree n given by the short exact sequence

$$0 \longrightarrow (n-1)\mathcal{O} \longrightarrow n\mathcal{O}(1) \longrightarrow \mathcal{O}(n) \longrightarrow 0\,, \tag{4.12}$$

which is obtained by putting $k = 0$ in (4.1). The sheaf $\mathcal{O}(n)$ breaks into the components on the MS-line, the decay being governed by (4.3). Hence, this is interpreted as a junction as in figure 4*b* with n strings connecting the $z = -1$ point and $n - 1$ strings connecting the $z = 1$ point to the vertex of the junction.

The resolutions corresponding to the spiral strings are obtained by forming Yoneda composites of complexes iteratively, as in the examples above. By putting $k = n - 2$ in (4.1) we derive

$$0 \longrightarrow \mathcal{O}(n-2) \longrightarrow 2\mathcal{O}(n-1) \longrightarrow \mathcal{O}(n) \longrightarrow 0\,. \tag{4.13}$$

For $n > 2$ the sheaves on the left are not stable in the strong coupling region $\mathfrak{S}$ rendering the sequence inappropriate for being interpreted as a junction. In order to obtain the projective resolutions suitable for our purpose we proceed iteratively, by finding a resolution of the leftmost sheaf in the above sequence (4.13), namely, $\mathcal{O}(n-2)$ by replacing n by $n - 2$ in (4.1) and putting $k = (n - 2) - 2$. This yields

$$0 \longrightarrow \mathcal{O}(n-4) \longrightarrow 2\mathcal{O}(n-3) \longrightarrow \mathcal{O}(n-2) \longrightarrow 0\,. \tag{4.14}$$

If $n \geq 4$, then this sequence again fails to correspond to a junction. We splice it with the sequence (4.13), deriving another projective resolution of $\mathcal{O}(n)$ as

$$0 \longrightarrow \mathcal{O}(n-4) \longrightarrow 2\mathcal{O}(n-3) \longrightarrow 2\mathcal{O}(n-1) \longrightarrow \mathcal{O}(n) \longrightarrow 0. \tag{4.15}$$

Continuing this and splicing the resulting resolutions to form Yoneda composites iteratively until we have a resolution ending of the left with either a single $\mathcal{O}$ or $\mathcal{O}(1)$, we arrive at a projective resolution of $\mathcal{O}(n)$ as

$$0 \longrightarrow \mathcal{O} \longrightarrow 2\mathcal{O}(1) \longrightarrow \cdots \longrightarrow 2\mathcal{O}(n-3) \longrightarrow 2\mathcal{O}(n-1) \longrightarrow \mathcal{O}(n) \longrightarrow 0 \tag{4.16}$$

if n is an even integer, and

$$0 \longrightarrow \mathcal{O}(1) \longrightarrow 2\mathcal{O}(2) \longrightarrow \cdots \longrightarrow 2\mathcal{O}(n-3) \longrightarrow 2\mathcal{O}(n-1) \longrightarrow \mathcal{O}(n) \longrightarrow 0 \tag{4.17}$$

if n is odd. Since all the sheaves participating in the resolution of $\mathcal{O}(n)$ are stable $\mathfrak{W}$, we can trade in the projective resolution for $\mathcal{O}(n)$ in $\mathfrak{D}^{\mathfrak{b}}(\mathcal{C})$. As in the examples above, the corresponding spiral string ends at $z = 1$ as $\mathcal{O}$ if n is even and at $z = -1$ as $\mathcal{O}(1)$ if n is odd. Since the large volume monodromy acts on a sheaf by tensoring with $\mathcal{O}(-2)$, the string spirals around $n/2$ times if n is even and $(n-1)/2$ times if n is odd, before ending up on the MS-line, in keeping with earlier findings [15].

Let us now discuss the transitions between the spiral strings and the junctions. So far we discussed the interpretation of the same dyon state $(1, n)$ with $n > 1$ as either a three-pronged junction or a spiral string ending at $z = \pm 1$. But at a certain point of the moduli space we expect a unique description of the dyons [26]. This means, in the moduli space the dyon is described by the junction in a certain region and by a spiral string in some other. On the boundary of these two regions there must be transitions between them [15].

Now, the replacement of an object C in $\mathfrak{D}^{\mathfrak{b}}(\mathcal{M})$ by its projective resolution, as given by (2.2) while always possible mathematically, is but physically meaningful when the charges of the branes are properly aligned to preserve some SUSY [18], as is encoded in the condition for Π-stability (2.5). This requires the grades of B and C of (2.2) to be equal. Hence, in order to determine which of the resolutions is to be chosen to replace $\mathcal{O}(n)$, among the two possibilities discussed above, we need to compare the grades of the objects in the resolutions. The appropriate resolution at a certain point in the z-plane is chosen depending on which of the resolutions satisfy (2.5). In order to see this explicitly, let us first rewrite the sequences (4.16) and (4.17) as

$$0 \longrightarrow \mathcal{O} \longrightarrow \mathcal{B} \longrightarrow \mathcal{O}(n) \longrightarrow 0\,, \tag{4.18}$$

and

$$0 \longrightarrow \mathcal{O}(1) \longrightarrow \mathcal{B} \longrightarrow \mathcal{O}(n) \longrightarrow 0\,, \tag{4.19}$$

respectively by denoting the intermediate complex by $\mathcal{B}$. These correspond to the spirals as discussed before. For all of these resolutions the grade of the complex $\mathcal{B}$ is

$$\varphi(\mathcal{B}) = -\infty\,, \tag{4.20}$$

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calculated from its Chern character $\mathrm{ch}(\mathcal{B}) = (0,n)$, $n > 1$, using the large volume expression (2.3). This is to be contrasted with the situation with the short exact sequence (4.12). The middle term is $n\mathcal{O}(1)$, with grade

$$\varphi\left(n\mathcal{O}(1)\right) = -3/2 \tag{4.21}$$

to the leading order in $\mathfrak{W}$. Now, the replacement of the sheaf $\mathcal{O}(n)$ in the derived category $\mathfrak{D}^{\mathfrak{b}}(\mathcal{C})$ by a projective resolution as in (2.2) with $C = \mathcal{O}(n)$ is physically proper if its grade equals that of B, as given by (2.5). For the spirals (4.18) and (4.19), $\phi(B) = -1$, while for the three-pronged junction (4.12) we have $\phi(B) = -3/2$. Thus, depending on whether the grade of $\mathcal{O}(n)$ is $-3/2$ or -1, the resolution appropriate for it to be replaced with is (4.12) or either of (4.16) or (4.17), if n is even or odd, respectively. This change of grade occurs as the dyon $(1,n)$ crosses the line $\arg(z) = \pi/2$ in the moduli space. Hence, as the dyon crosses this line, a spiral string corresponding to $\mathcal{O}(n)$, given by either of (4.16) and (4.17) makes transition into the junction given by (4.12), which agrees with earlier results [15]. These cases exhaust the possibilities that come from (4.1). Other values of k do not lead to qualitatively new resolution.

5. Conclusion

To conclude, in this article we have considered the dyons in the Seiberg-Witten theory from the point of view of B-branes wrapping holomorphic cycles in a Calabi-Yau manifold $\mathcal{M}$ in the geometrical engineering limit. Such B-branes correspond to the derived category of coherent sheaves on $\mathcal{M}$. In the geometric engineering limit this category is assumed to be generated by the coherent sheaves on the projective line, $\mathbb{P}^1$, the sole compact survivor of the limit [19]. The Π-stable sheaves in this derived category reproduce the dyons in the Seiberg-Witten theory [19]. The notion of the derived category stems from the need to identify an object of an abelian category with its resolutions. By introducing a grade in the derived category, the criterion of Π-stability provides a physical meaning to such replacements. The replacement is physically meaningful provided the grades of the objects participating in the resolution obey certain conditions (2.5). A dyon of the Seiberg-Witten theory decays on a certain marginal stability locus, the MS-line, according to this condition. The stable dyons on the two sides of the line are different. An estimation of the stable dyons on the two sides involve distinguished triangles in the triangulated category underlying $\mathfrak{D}^{\mathfrak{b}}(\mathbb{P}^1)$, which, in turn, correspond to short exact sequences in the abelian category. These triangles are interpreted as three-pronged junctions, involving prongs corresponding to strings on both sides of the MS-line resting on the stable dyons on the line itself. In general the prongs of a junction are found to have more than one strings in it, consistent with earlier findings [14]. Considering certain other projective resolutions of the invertible sheaves on $\mathbb{P}^1$, involving sheaves which are stable on only one side of the MS-line, the weak coupling region of the Seiberg-Witten theory, we show that such resolutions do not correspond to pronged junctions, as they are not allowed to penetrate the strong-coupling region beyond the MS-line. These resolutions, obtained by forming Yoneda composites of short exact sequences of sheaves on $\mathbb{P}^1$ iteratively, are interpreted as strings spiraling the MS-line in

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the weak coupling region, ending finally on 7-branes situated on the MS-line at $z = \pm 1$. Using the condition (2.5) we then compare the grades of the two resolutions and find that they may replace a dyon corresponding to an invertible sheaf $\mathcal{O}_{\mathbb{P}^1}(n)$, with $n > 1$ in but different regions of the moduli space spanned by z. As the dyon crosses the line $\arg(z) = \pi/2$ in the z-plane, a three-pronged junction takes over a spiral string or vice versa, thus corroborating earlier results [15].

These considerations, apart from reproducing the junctions and spirals of the Seiberg-Witten theory and their transitions, clearly brings out the role of the grade of Π-stability in deciding the replacement for an object in the derived category in a physically meaningful manner. It will be interesting to see if similar considerations predict such transitions in more complicated situations, for example, for string junctions corresponding to Seiberg-Witten theories arising from D-branes on del Pezzo surfaces. Investigation along this line is currently underway.

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